Basic Definitions for μSR Data (Handout for TSI2011)

August 10, 2011

Typical time differential (as opposed to time integrated) μSR data takes the form of the number of counts in each of an array of detectors in a time spectrum divided into bins of finite time width $N_i(t)$. The time t corresponds to the time difference between the muon's arrival and its decay as detected in the detector labelled i. Practically $N_i(t)$ is defined at only a discrete set of equally spaced time points t_j representing the midpoints of the set of time bins. $N_i(t)$ are often called the "(raw) histograms". Clearly the number of counts in the spectrum depends on the rate of incoming muons times and the overall time duration of the experiment.

Generally,

$$N_i(t) = N_{0i} e^{-t/\tau_\mu} [1 + A_i(t)] + B_i, \tag{1}$$

where N_{0i} is the overall normalization that increases with the number of muons used in the experiment. The radioactive decay of the muon with its lifetime $\tau_\mu = 2.19709(5)$ μs gives rise to the explicit exponential decay. The number of counts in each bin thus decreases exponentially with time, so that at times significantly longer than τ_μ, there will be relatively few counts in these bins as few muons survive to this venerable age. This also means that the data is statistically less significant at longer times. $A_i(t)$ is the *asymmetry (time spectrum)* for the detector and

$$A_i(t) = A_{0i} P_i(t),$$

where A_{0i} is the *(initial) asymmetry* for the detector, and $P_i(t)$ is the projection of the time dependent spin polarization of the muon $\vec{P}(t)$ projected onto the axis joining the beamspot's centre and the detector's central axis. As the muon spin is initially fully polarized $P_i(0) = 1$. $P_i(t)$ generally contains oscillating and relaxing terms that are the main object of the measurement.

Finally, B_i is a time independent background count rate. This term corresponds to spurious "stop" signals from background radiation as they are detected in the positron detectors, e.g. from cosmic rays, or more likely from the decay of muons stopped in the end of the beamline near, but not in, the spectrometer. Typically B_i is on the order of percent. With some difficulty one may construct an experiment where B_i is smaller than 1%.

The initial asymmetry, A_{0i} depends on many factors such as the muon decay anisotropy (the Michel parameters), the solid angle subtended by the detector, its overall detection efficiency (typically integrated over all possible positron energies), positron absorption in the sample and its surroundings, etc. For a typical experiment and detector A_{0i} is in the range of 0.2 to 0.4.

Rather than treating the individual histograms, the spectrometer usually consists of symmetric pairs of detectors around the sample. For ideally identical, symmetric opposing counters i and $-i$ (e.g. $i = L$ for left and $-i = R$ for right),

$$P_i = -P_{-i}. \tag{2}$$

The experimental "raw" asymmetry for the pair is defined by

$$A_{raw}(t) = \frac{(N_i(t) - B_i) - (N_{-i}(t) - B_{-i})}{(N_i(t) - B_i) + (N_{-i}(t) - B_{-i})}, \tag{3}$$

so this is basically the difference of the two count rates (normalized to their sum), but with the time-independent backgrounds subtracted. One important reason to use the asymmetry thus defined is that the exponential decay from the muon lifetime is automatically cancelled (provided the background subtraction is correct), so A_{raw} is proportional to $P_i(t)$, though still potentially offset from this due to imperfections in the symmetry of the detector pair.

To account for such imperfections, one usually forms the "corrected" asymmetry in the following way. In addition, to (1) above, we assume

$$N_{-i}(t) = N_{0(-i)} e^{-t/\tau_\mu}[1 + A_{-i}(t)] + B_{-i} \tag{4}$$

$$= \alpha N_{0i} e^{-t/\tau_\mu}[1 + \beta A_i(t)] + B_{-i}, \tag{5}$$

where this last equation defines the quantities $\alpha = N_{0(-i)}/N_{0i}$ and $\beta = A_{0(-i)}/A_{0i}$ where we assume (2) holds. For ideally symmetric counters, $\alpha = \beta = 1$. Commonly β is assumed to be 1, but α differs from 1. In this case, the corrected asymmetry is defined by

$$A_{corr}(t) = \frac{(N_i(t) - B_i) - (N_{-i}(t) - B_{-i})/\alpha}{(N_i(t) - B_i) + (N_{-i}(t) - B_{-i})/\alpha}. \tag{6}$$

In this case then,

$$A_{corr} = A_{0i} P_i(t),$$

i.e. it is proportional to the quantity of interest. Note that α is not known *a priori* and it depends on the detailed geometry of the experiment, so it is often fitted as a parameter in analysis. In some cases it is determined by a separate experiment. In fact, α determines the "baseline" or zero value for $A_{raw}(t)$. This can be seen by the general relation[1]

$$A_{raw} = \frac{(1-\alpha) + (1+\alpha\beta)A_{corr}}{(1+\alpha) + (1-\alpha\beta)A_{corr}},$$

or in inverted form

$$A_{corr} = \frac{(\alpha - 1) + (\alpha + 1)A_raw}{(\alpha\beta + 1) + (\alpha\beta - 1)A_raw}.$$

For $\beta = 1$ and for $\alpha \to 1$

$$A_{raw} = A_{corr} + \frac{1-\alpha}{1+\alpha}.$$

Considering that the spin polarization will relax towards a value of approximately zero, i.e. $P_i(t) \to 0$ as $t \to \infty$, it can be seen that α determines the long time limit of A_{raw}, while for a correct value of α, the long time limit of A_{corr} is zero. Sometimes the value of α is determined by a low transverse field measurement, where the full polarization oscillates about zero, so A_{raw} will oscillate about its finite baseline which can be determined by fitting. Once determined this value of α can be fixed. However, α generally may change with magnetic field (which alters the effective detector solid angles), temperature - as geometry will change slightly with thermal contraction, and with sample - due to positron absorption, so care must be taken in fixing α at a particular value, even when it is determined by such an α calibration experiment.

[1] see Noakes *et al.*, Phys. Rev. B **35**, 6597 (1987). The purists should check this algebra.